August 27-31, 2023
FLORENCE - ITALY
16th International Conference on Wind Engineering
ICWE 16

Non-lineal structural model of a suspension bridge for the reliability analysis considering uncertainties in wind speed and flutter derivatives

Juan Quintela [1], José Ángel Jurado [2], Arturo Fontán [3], Santiago Hernández [4]

[1] *School of Civil Engineering, Universidade da Coruña, Spain, juan.quintela.suarez@udc.es*
[2] *School of Civil Engineering, Universidade da Coruña, Spain, jose.angel.jurado@udc.es*
[3] *School of Civil Engineering, Universidade da Coruña, Spain, arturo.fontan@udc.es*
[4] *School of Civil Engineering, Universidade da Coruña, Spain, santiago.hernandez@udc.es*

SUMMARY:
The First Order Reliability Method was applied to bridge flutter instability considering the uncertainties of the flutter derivatives and extreme wind speeds. To calculate the probability of failure by flutter, the wind speed and each value of the flutter derivatives are considered as a random variable. The critical flutter speed is obtained by solving the dynamic equilibrium equation. The Hasofer-Lind algorithm was used to solve the reliability problem and was applied to a suspension bridge, using a non-lineal structural model made in OpenSees.

Keywords: Flutter, FORM, reliability

1. INTRODUCTION

Long-span bridges are some of the most challenging structures mankind has been able to build. They are remarkably slender and flexible; hence they are prone to suffer adverse effects caused by the wind. These kind of relevant constructions are possible nowadays due to two main facts: the appropriate mathematical formulations of wind induced phenomena as flutter or buffeting and the use of wind tunnel test to characterize the aeroelastic properties of bridges. Experiments with reduced models of full bridges allow observing the overall performance and sectional test of segments of bridge decks permit to obtain the aerodynamic coefficients and the flutter derivatives (FD) that will be used later on dynamic analysis under aeroelastic forces.

The flutter response of the bridge is significantly influenced by these experimental functions, which relate the aeroelastic forces with the displacement of the bridge deck. Small changes in their values could significantly alter the critical wind speed. Due to the experimental nature of the data as well as the identification technique utilized to extract each function, these flutter derivatives contain uncertainty. In fact, certain studies (Sarkar et al., 2009; Kusano et al., 2018) observed considerable discrepancies in the results of wind-related variables obtained by tests in wind tunnels. In this work, a reliability analysis of a suspension bridge working with the wind speed and the values of the flutter derivatives as random variables was carried out using the FORM method to obtain the probability of failure of the structure.

2. NON-LINEAL STRUCTURAL MODEL AND FLUTTER ANALYSIS

The finite element model of a suspension bridge and was made using free software OpenSees as shown in Fig. 1. OpenSees (McKenna, F. T., 1997) is an open-source, object-oriented software framework developed at UC Berkeley. The features of object-oriented programs make this software computationally efficient, flexible, extensible and portable.

The bridge deck, the towers and the main cable were modelled using OpenSees beam-column elements with large-displacements. An initial strain material is defined to apply the axial forces in the main cables. The hangers are modelled with truss elements, and they are connected to the deck elements by transversally rigid links.

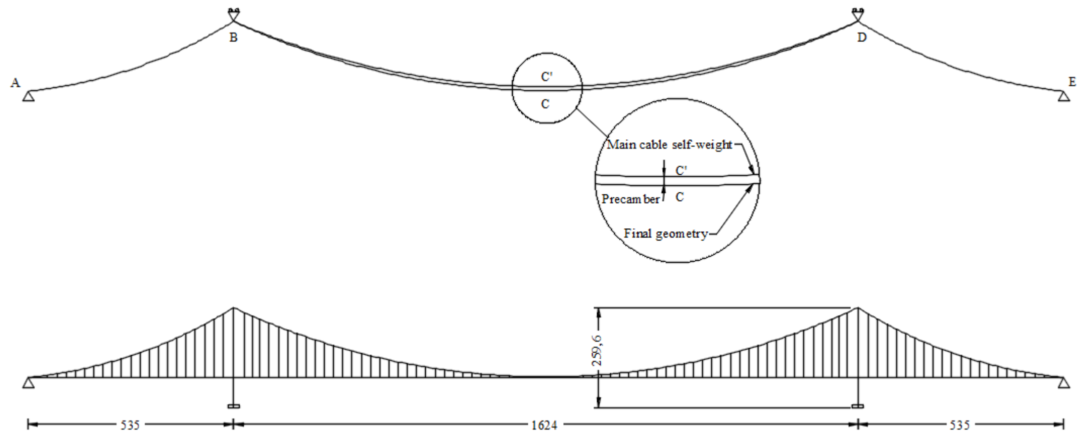

Figure 1. Suspension bridge scheme and main cable construction.

First, a structural model with only the elements of the main cables is used to obtain the final geometry using an iterative process. A precamber is assumed to obtain the position of the catenary and the initial stresses in the main cables. Then a non-linear static analysis with a load equal to the weight of the deck is carried out. The central vertical displacement is used as the precamber in the next iteration. This process finishes when the vertical displacement coincides with the precamber. The final structural model is defined with the last step position of the main cables considering the displacements and axial forces obtained and adding the rest of elements of the bridge; deck, hanger, links and towers. The natural modes and frequencies were obtained by a dynamic analysis taking into account second-order effects working with the final non-linear static analysis.

Flutter phenomena is analyzed using the dynamic equilibrium equation considering self-excited forces that can be written as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_a = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} \quad (1)$$

where **M**, **C**, **K** are the mass, damping and stiffness matrices and $\mathbf{C}_a$, $\mathbf{K}_a$ are the aeroelastic damping and stiffness matrices. Flutter derivatives relate the aeroelastic forces and the displacements of the deck and can be written as:

$$\boldsymbol{f}_a = \begin{Bmatrix} D_a \\ L_a \\ M_a \end{Bmatrix} = \frac{1}{2}U^2KB\begin{pmatrix} P_1^* & -P_5^* & -BP_2^* \\ -H_5^* & -H_1^* & BH_2^* \\ -BA_5^* & BA_1^* & B^2A_2^* \end{pmatrix}\begin{Bmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{Bmatrix} + \frac{1}{2}U^2K^2\begin{pmatrix} P_4^* & -P_6^* & -BP_3^* \\ -H_6^* & -H_4^* & BH_3^* \\ -BA_6^* & BA_4^* & B^2A_3^* \end{pmatrix}\begin{Bmatrix} v \\ w \\ \varphi_x \end{Bmatrix} \quad (2)$$

where A_i^*, H_i^* and P_i^* ($i=1,\dots,6$) are the flutter derivatives. By multi-modal analysis Eq. (1) leads to a non-linear eigenvalue problem as follows:

$$(\mathbf{A} - \mu\mathbf{I})\mathbf{w}_\mu e^{\mu t} = 0 \quad (3)$$

where matrix **A** depends on **K**, **C**, $\mathbf{K}_a$ and $\mathbf{C}_a$ matrices (Jurado and Hernández, 2001). This equation is solved as an eigenvalue problem using a multi-modal formulation (Jain et al., 1996; Jurado et al., 2011) where the critical flutter speed is obtained when the real part of one of the eigenvalues becomes null, indicating that the instability appears.

3. RELIABILITY WITH UNCERTAINTIES IN WIND SPEED AND FLUTTER DERIVATIVES

The first random variable considered in this study was the wind speed x_W because its probability function is generally known through a wind study. In addition, each point of the principal flutter derivatives ($A^*_{1\text{-}4}$, $H^*_{1\text{-}4}$) was considered as normal random variables with the mean being the value obtained in wind tunnel test. Fig. 2 shows one flutter derivative for which each point is considered as normal random variable.

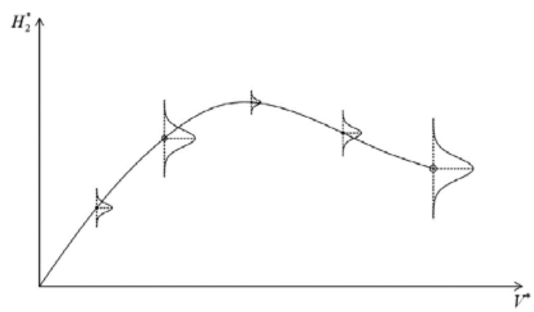

Figure 2. Flutter derivative expressed as function of several random variables.

With these random variables, the limit state function of the reliability analysis is defined as:

$$g(\mathbf{X}) = V_f(x_i) - x_W \quad i = 1,\dots,n \quad (4)$$

where V_f is the flutter wind speed, x_i are de n random variables that represent the set of point that define the flutter derivatives and x_W is the random variable of the wind speed.

The first-order reliability method (FORM) was proposed by Hasofer and Lind (Hasofer and Lind, 1974) and is formulated as the minimum distance between the origin in the normalized U-space to the failure surface g(**U**) = 0. The resulting optimization problem is:

$$\beta = min\sqrt{\mathbf{U}^\mathrm{T}\mathbf{U}} \ \ subject\ to\ \ g(\mathbf{U}) = 0 \quad (5)$$

where β is the reliability index, **U** are all the normalized random variables and the solution **U*** is denoted as the Most Probable Point of Failure (MPP).

4. RELIABILTY RESULTS AND CONCLUSSIONS

The reliability index was obtained using only one flutter derivative as a random variable, using the set of $A^*_{1\text{-}4}$ functions, the set of $H^*_{1\text{-}4}$ functions and the complete set of functions. For each case, a standard deviation of σ=15% and σ=30% of the mean value of the flutter derivative was considered. The wind velocity was described as an equivalent normal distribution $x_W = N(41.249; 3.033)$. The results obtained are shown in Table 1.

Table 1. Reliability results and probability of failure.

Random variables	Nº of variables	$\sigma = 0.15$		$\sigma = 0.30$	
		β	P_f	β	P_f
x_W, A^*_1	4	14.91	1.38E-50	8.67	2.15E-18
x_W, A^*_2	5	11.45	1.04E-30	3.77	7.97E-05
x_W, A^*_3	6	15.84	7.18E-57	15.79	1.61E-56
x_W, A^*_4	6	16,32	3.48E-60	15.56	6,41E-55
x_W, H^*_1	5	16.50	1.73E-61	16.26	8.15E-60
x_W, H^*_2	6	16.65	1.51E-62	16.58	4.26E-62
x_W, H^*_3	5	15.21	1.41E-52	7.09	6.38E-13
x_W, H^*_4	6	16.50	1.70E-61	16.04	3.29E-58
$x_W, A^*_1, A^*_2, A^*_3, A^*_4$	18	6.62	1.76E-11	3.14	3.20E-04
$x_W, H^*_1, H^*_2, H^*_3, H^*_4$	19	14.16	7.18E-46	6.30	1.41E-10
x_W, A^*, H^*	37	5.40	3.33E-08	2.72	3.20E-03

Results show that the probability of failure is higher when the standard deviation is increases. In addition, it can be seen that the A^*_2 function has the most impact in the reliability index, obtaining a considerably lower β compared to the other sets of functions. When the complete set of flutter derivatives is used, the probability of failure increases significantly as more random variables are introduced in the analysis.

ACKNOWLEDGEMENTS

This research has been funded by the Spanish Ministry for Science and Innovation, in the frame of the research project PID2019-110786GB-I00.

REFERENCES

Hasofer, A. M., Lind, M. C., 1974. An Exact and Invariant First Order Reliability Format. Journal of Engineering Mechanics Division, Vol. 100, 111–121.

Jain, A., Jones, N.P., Scanlan, R.H., 1996. Coupled flutter and buffeting analysis of long-span bridges. Journal of Structural Engineering. Vol. 122, 7, 716–725.

Jurado J. A., Hernández, S. 2001. Sensitivity analysis of bridge flutter with respect to mechanical parameters of the deck. Structural and Multidisciplinary Optimization. 27, 272-283.

Jurado, J. A., Hernandez, S., Nieto, F., Mosquera, A., 2011. Bridge Aeroelasticity. Sensitivity Analysis and Optimal Design. WITPress, Southampton, UK.

Kusano, I., Baldomir, A., Jurado, J.A., Hernández, S., 2018. The importance of correlation among flutter derivatives for the reliability based optimum design of suspension bridges. Engineering. Structures. 173, 416–428.

McKenna, F. T., 1997. Object-Oriented Finite Element Programming: Frameworks for Analysis, Algorithms and Parallel Computing. PhD thesis, University of California, Berkeley.

Sarkar, P., Caracoglia, L., Haan, Jr. F., Sato Murakoshi H., 2009. Comparative and sensitivity study of flutter derivatives of selected bridge deck sections, Part 1: analysis of inter-laboratory experimental data. Engineering Structures. 31, 158–169.